

Intercalibration of dIdD and Fluxgate Magnetometers

Balázs HEILIG

Eötvös Loránd Geophysical Institute, Tihany Geophysical Observatory
Kossuth u. 91, H-8237 Tihany, Hungary
e-mail: heilig@elgi.hu

Abstract

Fluxgate magnetometers are the most frequently used vector magnetometers in current observatory practice. However, at more and more sites, alternative solutions, e.g. dIdD magnetometers are also applied to record the geomagnetic variation. In this paper we demonstrate the advantage of the simultaneous use of a fluxgate and a dIdD by introducing a cross-calibration between these instruments. The calibration of a fluxgate is a process to determine 9 parameters including 3 scale factors, 3 offset values, 3 orthogonality parameters. As follows from the dIdD principles, the dIdD is void from offset and scale factor errors. Therefore, to calibrate the dIdD one has to determine one single orthogonality error; the other orthogonality conditions are satisfied automatically since the 3rd dIdD axis is defined mathematically as perpendicular to the two physical axes of the instrument. Furthermore, the intercalibration can yield information on relative orientation errors, i.e. the orientation of one instrument can be determined relative to a well-oriented reference instrument.

1. Introduction

Fluxgate magnetometers are the most frequently used vector magnetometers in current observatory practice. However, at more and more sites, alternative solutions, e.g. dIdD magnetometers are also applied to record the components of the geomagnetic variation. Three component fluxgate variometers have 12 parameters to be determined during a calibration process: 3 scale values, 3 offsets, 3 orthogonality errors and 3 orientation errors (see Table 1). At the same time, the dIdD has only 4 parameters needed to be calibrated: the orthogonality of the two coil-system (ε_{DI}), and the 3 orientation angles (I_0 , D_0 and ε_0). The dIdD can have neither scale value, nor offset problems since the vectors of the magnetic field components in the arbitrary reference frame of this instrument are derived from absolute, total field readings. Thus, the dIdD can be considered as an absolute instrument, but, unfortunately, only in its own refer-

ence frame. The problem of calibration of the dIdD equals to finding its (maybe non-orthogonal) reference frame.

We demonstrate here a process to calibrate the dIdD against a reference fluxgate magnetometer supposed to be sufficiently oriented. The dIdD calibrated according to the method described below can be used as a vector variometer (not as an absolute instrument). Moreover, the scale values of the reference magnetometers can be determined during the same procedure.

2. Computation of the dIdD Components (The dIdD Reference Frame)

First let us suppose that the dIdD is perfectly aligned and oriented, i.e. its I and D coil axes are orthogonal, the D coil axis is horizontal and both axes are perpendicular to the magnetic field vector. The third axis, let us call it the S (sensor) axis, is defined mathematically to be perpendicular to both coil axes, so taking into account our assumptions it is parallel to the magnetic field. Moreover, the S axis is directed parallel to the field, while the D coil axis is directed toward magnetic east, and the I coil is directed so that the S-D-I axes form a right-handed system. This S-D-I system is referred to as the dIdD reference frame in this paper. The same notation will be used also for an arbitrary orientation of the instrument, the dIdD reference frame is always defined physically by the orientation of the coils (Schott and Pankratz 2001).

The dIdD components in an arbitrary dIdD reference frame can be computed as (Schott *et al.* 2001):

$$B_i = \frac{I_+^2 - I_-^2}{4A_i}, \quad (1)$$

$$B_d = \frac{D_+^2 - D_-^2}{4A_d}, \quad (2)$$

$$B_s = \sqrt{F^2 - B_i^2 - B_d^2}, \quad (3)$$

$$A_i = \sqrt{(I_+^2 + I_-^2 - 2F^2)/2}, \quad (4)$$

$$A_d = \sqrt{(D_+^2 + D_-^2 - 2F^2)/2}, \quad (5)$$

where F (total field), I_+ (deflected parallel to the I axis), I_- (deflected antiparallel to the I axis), D_+ (deflected parallel to the D axis), D_- (deflected antiparallel to the D axis) are the five readings of a dIdD sequence, A_i and A_d are the deflection fields generated by the I- and D-coils, respectively, B_i , B_d , and B_s are the dIdD vector components along the I, D, and S axes. Here we supposed that the opposite deflection fields applied in the same coil have the same magnitude, and that the field variation is negligible during the measurement sequence. Current dIdD models are based on the GSM-19 Overhauser magnetometer model of the Gem Systems and able to take a whole measurement cycle during a few seconds.

3. Transformation from the dIdD to the XYZ Reference Frame

Unlike the usual treatment we do not use the approximative formulae to calculate dD and dI (e.g. Pankratz *et al.* 1999). The transformation of the data from the dIdD reference frame to the XYZ reference frame is achieved by correcting the orthogonality error and implementing a 3D rotation represented here as three consecutive rotations around the S, D and I axes, respectively.

1. The orthogonality error ε_{ID} , that is taken positive if the angle between the I and D axes is less than 90° , can be removed e.g. by the following formulae:

$$B_{ic} = B_i, \quad (6)$$

$$B_{dc} = \frac{B_d - B_i \sin(\varepsilon_{ID})}{\cos(\varepsilon_{ID})}, \quad (7)$$

$$B_{sc} = \sqrt{B_i^2 + B_d^2 + B_s^2 - B_{ic}^2 - B_{dc}^2}, \quad (8)$$

where B_{ic} , B_{dc} and B_{sc} stand for the corrected components. This transformation sets the D axis perpendicular to the I axis.

2. In the next step the orthogonal reference frame is rotated by ε_0 about the S axis in a clockwise direction when looking towards the origin. This step corresponds the leveling of the D coil. The matrix of rotation is:

$$\overline{\overline{E}}_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon_0) & -\sin(\varepsilon_0) \\ 0 & \sin(\varepsilon_0) & \cos(\varepsilon_0) \end{pmatrix}. \quad (9)$$

3. A further rotation by I_0 about the D-axis, in a counterclockwise direction when looking towards the origin, brings the I axis horizontal. The resulting reference frame corresponds to an HDZ system:

$$\overline{\overline{I}}_0 = \begin{pmatrix} \cos(I_0) & 0 & -\sin(I_0) \\ 0 & 1 & 0 \\ \sin(I_0) & 0 & \cos(I_0) \end{pmatrix}. \quad (10)$$

4. Finally a rotation by D_0 about the I axis, in a clockwise direction when looking towards the origin, transforms the HDZ system to a geographic XYZ system.

$$\overline{\overline{D}}_0 = \begin{pmatrix} \cos(D_0) & -\sin(D_0) & 0 \\ \sin(D_0) & \cos(D_0) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (11)$$

The 3D rotation can be written in the form:

$$\vec{B}_{xyz} = \overline{\overline{D}}_0 \left(\overline{\overline{I}}_0 (\overline{\overline{E}}_0 \cdot \vec{B}_{sdi_corr}) \right), \quad (12)$$

where B_{sdi_corr} denotes the components in the orthogonalized coordinate system, i.e. (B_{sc}, B_{dc}, B_{ic}) , and B_{xyz} stands for the XYZ representation of the field vector.

4. Calculation of D_0 and I_0 from a Single Absolute Measurement

Supposing the orthogonality of the coil axes ($\varepsilon_{ID} = 0$), the transformation can be unfolded from Eq. (12) as:

$$X = B_{sc} \cos D_0 \cos I_0 - B_{dc} (\cos D_0 \sin I_0 \sin \varepsilon_0 + \sin D_0 \cos \varepsilon_0) - B_{ic} (\cos D_0 \sin I_0 \cos \varepsilon_0 - \sin D_0 \sin \varepsilon_0), \quad (13)$$

$$Y = B_{sc} \sin D_0 \cos I_0 - B_{dc} (\sin D_0 \sin I_0 \sin \varepsilon_0 - \cos D_0 \cos \varepsilon_0) - B_{ic} (\sin D_0 \sin I_0 \cos \varepsilon_0 + \cos D_0 \sin \varepsilon_0), \quad (14)$$

$$Z = B_{sc} \sin I_0 + B_{dc} \cos I_0 \sin \varepsilon_0 + B_{ic} \cos I_0 \cos \varepsilon_0. \quad (15)$$

If we know the absolute values of the field components for a certain time and also the simultaneous SDI components measured by the dIdD, then using numerical methods we can determine the three orientation angles, ε_0 , D_0 , and I_0 from Eqs. (13)-(15).

If we go further supposing that the D axis is horizontal ($\varepsilon_0 = 0$), D_0 and I_0 can be expressed in closed form:

$$\cos I_0 = \frac{ZB_i + \sqrt{B_s^2(B_s^2 + B_i^2 - Z^2)}}{B_i^2 + B_s^2}, \quad (16)$$

$$\cos D_0 = \frac{XW + \sqrt{X^2W^2 - (W^2 + B_d^2)(X^2 - B_d^2)}}{W^2 + B_d^2}, \quad (17)$$

$$\text{where} \quad W = B_s \cos I_0 - B_i \sin I_0. \quad (18)$$

This method yields a simple alternative way to estimate the dIdD reference frame, with the same assumptions, of the solution proposed by Schott and Pankratz (2001).

5. The Intercalibration Process

Equations (16)-(18) are valid only when two assumptions are fulfilled: the D-axis is horizontal and the two coils are orthogonal. The intercalibration process described below is an example for a procedure that is suitable for determining the missing error values as well. The calibration process benefits from the natural variation of the geo-magnetic field. The method is based on minimizing the fluctuation in the component differences between the dIdD to be calibrated and the reference vector magnetometer.

The difference of the corresponding vector components, e.g. $X_1 - X_2$ of two properly aligned and oriented (i.e. calibrated) variometers, apart from some noise, gives a

time independent, constant value. Variometers with different orientations, however, give fluctuating component differences. Minimizing the fluctuations in all components simultaneously while changing the calibration parameters one can estimate the mutual orientation, the relative scale values and/or the relative misalignment errors of the two instruments. Finally, as an additional information we can calculate the baseline differences between the calibrated and the reference instrument.

The intercalibration of a dIdD and a fluxgate instrument is especially advantageous. Since all the scale values of the dIdD equal 1 (Table 1), the relative scale values to be determined will be the actual scale values of the fluxgate instrument. An important consequence of the mathematical definition of the S axis of the dIdD is that this instrument has only one possible orthogonality error, namely the misalignment of the mutual position of the two coils, ε_{ID} . Assuming that the reference triaxial fluxgate sensors are orthogonal and that these sensors are orientated perfectly in the geographic XYZ frame, the calibration will give an estimation of the orthogonality error and the three orientation angles of the dIdD. The baseline differences between the two instruments in this case will be an estimation of the actual fluxgate baselines.

Table 1
Comparison of dIdD and fluxgate calibration parameters

Calibration parameters	Fluxgate	dIdD	Absolute vector magnetometer
Scale values	c_x, c_y, c_z	1, 1, 1	1, 1, 1
Baseline	X_b, Y_b, Z_b	0, 0, 0	0, 0, 0
Orthogonality errors	3 angles	0, 0, ε_{ID}	0, 0, 0
Orientation	3 angles	ε_0, D_0, I_0	0, 0, 0

In practice, there does not exist any perfect reference magnetometer. Both the sensor misalignment and orientation errors can reach tens of arc minutes. However, we can still want to determine the 4 dIdD calibration parameters needed to convert the dIdD data into the reference frame of the fluxgate magnetometer, e.g. when we intend to fill data gaps of the primary recording system with data derived from the measurements of a poorly oriented dIdD.

During the optimization process a minimum is searched for the sum of the *rms* values (R) of the three detrended difference signals:

$$R = \sum_{i=x,y,z} rms \left[filter \left(B_i^{didd}(t) - B_i^{fg}(t) \right) \right], \quad (19)$$

where

$$B_i^{didd}(t) = \overline{\overline{D_0}} \left[\overline{\overline{I_0}} \left(\overline{\overline{E_0}} \cdot \vec{B}_{sdi_corr}(t) \right) \right]_i, \quad i = x, y, z. \quad (20)$$

The 7 free parameters are: I_0 , D_0 , ε_0 , ε_{DI} for the dIdD, and c_x , c_y , c_z , for the fluxgate. Before calculating the rms values, two filters had been applied to the difference signals: a highpass filter to exclude the slow trends connected to temperature variations of the fluxgate magnetometer, and a lowpass filter to remove the ‘high’ frequency instrument noise. The computation was implemented on a daily basis using a Quasi-Newton method in Matlab’s Optimization Toolbox (function *fminunc*). The starting values of I_0 and D_0 were calculated from Eqs. (16)-(18). All other initial angle parameters were set to zero and all scale values to 1.

In the example given below we used the DMI FGE fluxgate, the primary variometer of the Tihany Observatory as a reference instrument to find the orientation of a first generation (i.e. not suspended model with a coil system 3 dm in diameter) dIdD that was installed in February 2000. During the installation process the D axis was set horizontal within 10 arc minutes, and also the orthogonality of the coils was adjusted with the same precision. The initial values of $I_0 = 63.398^\circ$ and $D_0 = 2.401^\circ$ were derived from a set of absolute measurements. Since then the raw XYZ components of the dIdD have been recorded, i.e. for the calculation of components not the actual but the initial orientation parameters were used. However, during the passed years the dIdD pretty changed its position. Using the method presented here we implemented the intercalibration procedure for all (available) days in 2003.

In Fig. 1 a and b the component differences between dIdD and FGE before and after the calibration process are shown for August 23. It can be seen that the rather large amplitude fluctuations in the raw differences almost completely disappeared after the calibration ($R_{min} = 0.18$ nT). The accuracy of the parameter estimation mainly depends on the overall noise level in the data, more exactly on the signal-to-noise ratio in the difference time series. For example supposing a variation of 30 or 300 nT and a 0.3 nT noise level, an 1% or 0.1% relative accuracy can be achieved, respectively. The effect of noise can be somewhat decreased by filtering the data, or averaging the results for several days.

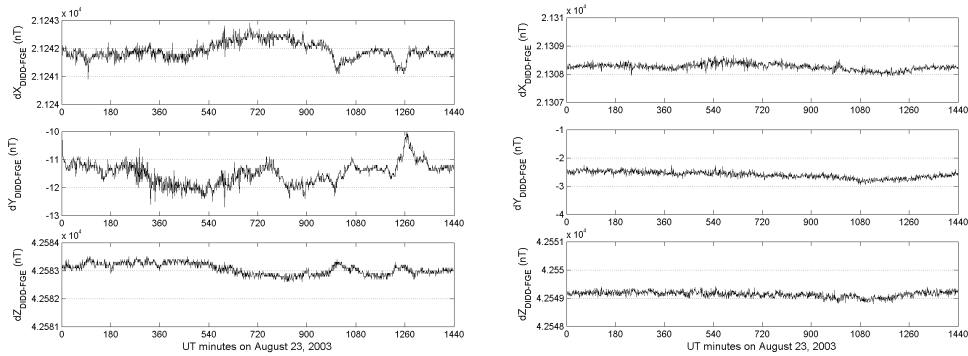


Fig. 1. The component differences between dIdD and FGE during one day (a) before, and (b) after the calibration process.

In Fig. 2 the deviations of the estimated dIdD calibration parameters from their yearly means are plotted for the whole year. The yearly mean values ($\pm$ standard

deviations) calculated for all days (when R_{min} was less than 0.3 nT) are: $I_0 = 63.276^\circ \pm 0.099^\circ$, $D_0 = 2.717^\circ \pm 0.209^\circ$, $\varepsilon_0 = -0.664^\circ \pm 0.234^\circ$, $\varepsilon_{DI} = -0.101^\circ \pm 0.146^\circ$, $c_x = 0.9983 \pm 0.0015$, $c_y = 0.9989 \pm 0.0016$, $c_z = 0.9976 \pm 0.0070$. We note here that according to the calibration certificate of the FGE instrument all the misalignments of fluxgate sensors are less than 1 mrad. With the help of the resulting 4 transformation parameters, the dIdD SID data can be converted to an approximate XYZ system applying Eqs. (6)-(11), in addition the FGE baseline can also be estimated. Of course the converted dIdD components will not point in the true XYZ directions since the reference FGE was obviously not perfectly oriented and not completely void from orthogonality errors. Even so, the baseline calculation for e. g. November 12 resulted values very close to the true adopted baselines ($X_b = 21292$ nT, $Y_b = -4$ nT, $Z_b = 42564$ nT, the difference from the adopted baselines are 38 nT, 4 nT and 10 nT, respectively). It means that the FGE was installed carefully and served as a really good reference.

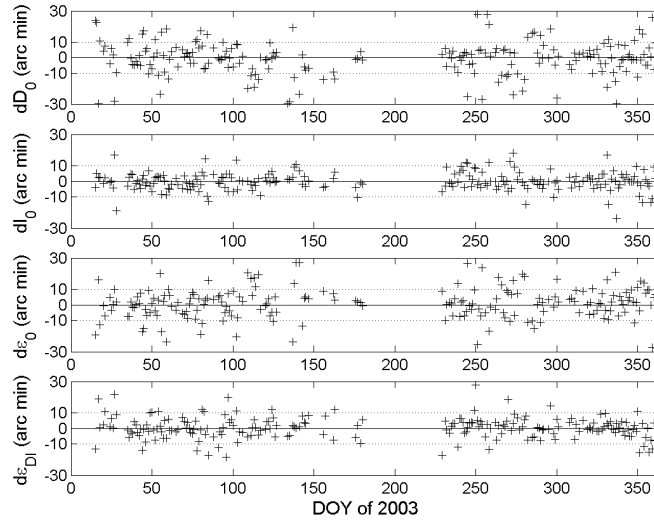


Fig. 2. The deviation of the estimated dIdD calibration parameters from their yearly means.

Here our intention was to implement a relative calibration between the dIdD and the main recording system of the observatory to be able to fill incidental data gaps. For this purpose we did not need a perfect calibration, we could be satisfied with a ‘good enough’ solution, when the component difference fluctuations do not surpass the instrument noise level. The real dIdD orientation can be found in a similar way. In that case instead of using a reference variometer, a sufficiently long series of absolute measurements is needed as a reference.

6. Summary

In this paper a method was presented for the determination of the orientation of a dIdD instrument, as well as its orthogonality error. The process is based on minimiz-

ing the fluctuation of the component differences between the calibrated and the reference instruments. The accuracy of this method at mid-latitudes is at the order of a few mrad, but even better can be achieved by averaging the estimations for 10-20 days. The accuracy level is limited by the S/N of the difference signals.

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References

- Pankratz, L.W., E.A. Sauter, A. Körmendi and L. Hegymegi, 1999, *The US-Hungarian Delta I – Delta D (dIdD) Quasi-absolute Spherical Coil System. Its history, evolution and future*, Geophysical Transactions **42**, 195-202.
- Schott, J.J., and P. Leroy, 2001, *Orientation of the dIdD magnetometer*, Contributions to Geophysics and Geodesy **31**, 43-50.
- Schott J.J., V. Boulard, A. Péres, J.M. Cantin and J. Bitterly, 2001, *Magnetic component measurement with the dIdD*, Contributions to Geophysics and Geodesy **31**, 35-42.

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