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An Assessment of the BGS δDδI Vector Magnetometer

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Abstract

Assuming optimal conditions, the δDδI vector magnetometer can be considered as a semi-absolute instrument. In this paper, the real situation and its differences with the ideal one are critically examined and regarded as potential sources of error. The analysis is applied to the equipment designed by the British Geological Survey (BGS) in the late 1980s, and the given figures are provided from the instrument in use at the Livingston Island Geomagnetic Observatory (LIV). Improved versions have been developed since then, being based on generally the same principles of measurement; thus, most of the results shown can be adapted to the new generation of magnetometers of this type.

1. Introduction

The automatic BGS δDδI vector magnetometer (Riddick *et al*. 1995) consists of two mutually perpendicular pairs of Helmholtz coils and a proton magnetometer mounted at their center for total field measurements, *F*. Two bias currents are alternatively applied to each pair of coils in opposite senses, generating additional magnetic fields in the proton magnetometer, which allow the measurement of variations in magnetic Declination (from the D pair), δD, and Inclination (from the I pair), δI. This device was initially conceived as a semi-absolute instrument (Alldredge 1960, or Alldredge and Saldukas 1964). Following this assumption, the origin of the angular variations, D_0 and I_0, requires to be established once by making absolute measurements; after that, D and I are computed from $D_0 + \delta D$ and $I_0 + \delta I$, respectively. For this equipment to operate as a semi-absolute observatory instrument, a series of requirements are necessary, the most important of which are presented below:

– Stability of the pier where the instrument is deployed;
– Both D coils are parallel and contained in a vertical plane;
– The D and I pairs are perfectly, mutually perpendicular;

– The coils polarization is identical in both (direct and inverse) directions;
– Geometrical stability (e.g., no temperature or humidity effects);
– The proton magnetometer is located at the centre of both pairs and its dimensions are small as compared with the coils.

If these requisites are fulfilled, it can be shown that Eqs. (1) and (2) hold:

$$D = D_0 + \delta D = D_0 + \arcsin\left(\frac{f(D)}{\cos I}\right), \tag{1}$$

$$I = I_0 + \delta I = \arcsin\left(\sin I_0 \sqrt{1 - f(I)^2 - f(D)^2} + f(I)\cos I_0\right), \tag{2}$$

where:

$$f(x) \equiv \frac{F_{x+}^2 - F_{x-}^2}{4FA_x}, \qquad x = D, I \tag{3}$$

$$A_x = \sqrt{\frac{F_{x+}^2 + F_{x-}^2}{2} - F^2} \tag{4}$$

and $F_{x\pm}$ is the magnetic field intensity generated by the direct (+) and inverse (-) polarization of the corresponding coils. In practice, A_x is approximately the mean value of the direct and inverse fields, $A_{x\pm}$, generated by the coils along their respective axes, while $F_{x\pm}$ result from the vector addition of $A_{x\pm}$ and the natural magnetic field.

2. The Real Case

In this section we analyze the errors produced in a real situation, as operationally not all of the above mentioned requirements can be met.

2.1 Approximations

Developing Eqs. (1) and (2) up to second order we get:

$$D = D_0 + \delta D \cong D_0 + \frac{180^\circ}{\pi}\frac{f(D)}{\cos I} \cong D_0 + \frac{180^\circ}{\pi}\frac{f(D)}{\cos I_0}\left(1 + f(I)\tan I_0\right), \tag{5}$$

$$I = I_0 + \delta I \cong I_0 + \frac{180^\circ}{\pi} f(I) - \frac{90^\circ f^2(D)}{\pi}\tan I_0, \tag{6}$$

where D and I (and related angles) are expressed in degrees.

In the observatory practice, Eq. (5) is normally used integrally, while the first-order approximation is used for Eq. (6). The term neglected in this equation can be written as follows:

$$\frac{90^\circ f^2(D)}{\pi}\tan I_0 \cong \frac{\pi}{360^\circ}\sin I_0 \cos I_0 \delta D^2. \tag{7}$$

The error generated as a result of omitting this term depends on the change in Declination, δD, which can easily exceed 1º during periods of high magnetic disturbance. These field changes can lead to an error of the order of 15 arc-seconds at mid-latitude observatories. We can even consider that the absolute value of δD is increasing with time in a given observatory as a result of the secular variation; in consequence, if the instrument is not re-oriented from time to time in order to reduce δD, the error generated by neglecting the last term of Eq. (6) can reach the order of 0.1 arc-minutes per year at locations of a high secular variation. This error can be overcome by simply using the second-order expression, thus avoiding the annoying necessity of re-orientating the coils.

2.2 Accuracy and time resolution

The maximum theoretical accuracy can be determined by assuming that the only limitation of the instrument is due to the proton magnetometer precision, acting in a random manner for the different measurements of the field vectors made during a cycle of measurements, i.e., F_{D+}, F_{D-} and F for D; F_{I+}, F_{I-} and F for I. Hence, the associated statistical error is propagated as a sum of squares for both, D:

$$\delta(\delta D) = \sqrt{\left(\frac{\partial(\delta D)}{\partial F_{D+}}\delta F_{D+}\right)^2 + \left(\frac{\partial(\delta D)}{\partial F_{D-}}\delta F_{D-}\right)^2 + \left(\frac{\partial(\delta D)}{\partial F}\delta F\right)^2}$$

$$= \frac{180^{\circ}}{\pi}\frac{\delta F}{\cos I_0}\sqrt{\frac{1}{2A_D^2}+\frac{1}{2F^2}} \tag{8}$$

and I (where the terms A_D and cos I_0 from Eq. (8) are to be substituted with A_I and 1, respectively). With a series of assumptions, and considering that the accuracy of the proton magnetometer is $\delta F = 0.2$ nT in its full range, an accuracy between 1 and 5 arc-seconds can be obtained for most of the mid-latitude observatories for both angular elements. In practice, accuracy will also depend on the stability of the current applied to the coils in both senses, $A_{x\pm}$.

The time resolution is limited by the length of time it takes to make a complete set of measurements. The time required for total field measurements depends upon the proton magnetometer type in use; additionally, both F_{x+} and F_{x-} need a certain time to establish a stable current in the coils. At LIV, the complete cycle (F_1, F_{I+}, F_{I-}, F_2, F_{D+}, F_{D-}) takes for 28 s; this slow sampling impedes the detection of rapid magnetic variations. The new generation of δDδI magnetometers has reduced the cycle duration in order to increase the sampling rate and meet INTERMAGNET requirements (see www.intermagnet.org/im_manual.pdf).

2.3 Instrument orientation and imperfect orthogonality of the coils

Poor instrument orientation is another factor that limits its accuracy. This misorientation can be described by three successive rotations defined by the angles Φ, ψ and θ around the three coordinate axes. If D_0 and I_0 are the Declination and Inclination of the D and I coils respectively, let Φ be the difference between the real value of D_0 and its assumed value, ψ the difference between the assumed value for I_0 and the corresponding real value, and θ the angle between the vertical plane and the plane defined by the D coils (θ is positive when the D coils are turned anti-clockwise when looking from the North side). It can be demonstrated that Eqs. (1) and (2) are modified as follows as a result of this general rotation:

$$D = D_0 + \varphi + \arcsin\left(\frac{f(D) - \sin\theta\sin I}{\cos\theta\cos I}\right), \tag{9}$$

$$\begin{aligned} I = {} & \arcsin\Big[\sin(I_0 - \psi)\cos\theta\sqrt{1 - f(I)^2 - f(D)^2} + f(D)\sin\theta \\ & + f(I)\cos\theta\cos(I_0 - \psi)\Big]. \end{aligned} \tag{10}$$

It should be noted that angles Φ and ψ are corrected with the D and I base lines, respectively, but not θ. Hence, it is very important to align the D coils in a vertical plane when the instrument is deployed for the first time. In order to avoid this effect, the new generations of δDδI include suspended coils (Hegymegi 2004). The above expressions can also be used to characterize tilting of the pier; in this case, we have to consider the possibility that Φ, ψ and θ are time-dependent.

The imperfect orthogonality of the D and I pairs of coils can also be analyzed. Let σ be the angle between the plane defined by the D coils and the line orthogonal to the I coils (σ is positive when the I coils are turned anti-clockwise when looking from the North side). It can be shown that Eq. (9) still holds, whilst a term $\sigma \cdot f(D)$ is to be added to the second-order development of Eq. (10).

2.4 Errors due to current instabilities. Despiking

A fundamental requirement for the correct measurement of the D and I variations is that both deflecting fields, A_{x+} and A_{x-}, are of equal magnitude. Let us call it "hypothesis of symmetrical polarization". If this hypothesis is not fulfilled, an erroneous measurement will be produced, resulting in a spike in the data. The fundamental problem arises from our ignorance of the separated values A_{x+} and A_{x-}. The technique we propose following is aimed at detecting spikes from the δDδI data. It consists of searching for asymmetrical polarizations. In practice, the current stability varies, as it can be observed from Fig. 1, where a temperature dependence is clearly shown. Nevertheless, this effect is not in general appreciated during the short time a single measurement cycle lasts.

Similar reasonings apply to both Declination and Inclination; hence, let us consider only Inclination. From geometrical considerations, and with the above mentioned approximations, it is easy to compute δI for both the direct and inverse polarizations:

$$\delta I_+ = \frac{180^\circ}{\pi}\left(\frac{F_{I+}^2 - F - A_{I+}^2}{2FA_{I+}}\right), \tag{11}$$

$$\delta I_- = -\frac{180^\circ}{\pi}\left(\frac{F_{I-}^2 - F - A_{I-}^2}{2FA_{I-}}\right). \tag{12}$$

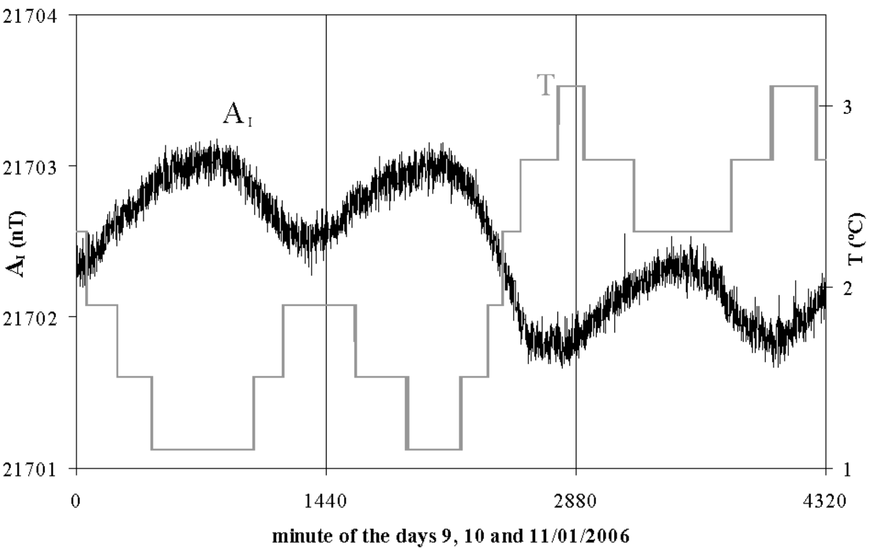

Fig. 1. The field intensity A_I (Eq. 4, black line in the figure) generated by the bias currents in the I coils at LIV is negatively correlated with temperature (grey line).

Additionally, if we make $A_{I+} = A_{I-} \equiv A_I$ and $\delta I_+ = \delta I_- \equiv \delta I$, we obtain Eq. (4) for A_I and the first-order approximation of Eq. (6) for δI, i.e., $180º/\pi \cdot f(I)$. By similitude with Eqs. (11) and (12), let us now introduce the following two observables:

$$\delta I'_+ = \frac{180º}{\pi}\left(\frac{F_{I+}^2 - F - \langle A_I \rangle^2}{2F\langle A_I \rangle}\right), \tag{13}$$

$$\delta I'_- = -\frac{180º}{\pi}\left(\frac{F_{I-}^2 - F - \langle A_I \rangle^2}{2F\langle A_I \rangle}\right), \tag{14}$$

where $<A_I>$ stands for the "normal" or mean value of the polarization intensity, and it can be computed in several ways, as for example:

$$\langle A_I \rangle = \frac{1}{60}\sum_{\substack{m'=m-30 \\ (m'\neq m)}}^{m+30} A_{I,m'}\ , \tag{15}$$

i.e., the mean of A_I computed as Eq. (6) one hour around the minute of interest, m, excluding the value corresponding to this minute. If an anomaly is present in the minute m, e.g., only in the direct (+) polarization, it will be revealed in the $\delta I'_+$ observable (Eq. 13) due uniquely to the anomalous value of F_{I+}, since $<A_I>$ is independent of minute m; on the contrary, if the inverse polarization (and hence F_{I-}) is normal, $\delta I'_-$ (Eq. 14) will have the correct value. Therefore, a plot of $\delta I'_+$ and $\delta I'_-$ against time in the same graph will help to identify spikes in the δDδI record. If both values coincide within the instrument accuracy (see subsection 2.2), there is no reason to suspect of the value obtained for δI; otherwise, the measurement may be in error. The discrepancy between the δI values obtained from both polarizations will become more evident if their difference is plotted, giving rise to a new observable: $disI \equiv (\delta I'_+ - \delta I'_-)/2$. In fact, this observable allows us to easily detect anomalous polarizations, since it can be shown that (for minute m) $disI_m \approx 180^\circ/\pi \cdot (A_{I,m} - \langle A_I \rangle)/F$, but it does not provide any information on the actual accomplishment of the "symmetry hypothesis", which is the only valid criterion to reject any data. In order to do this, *ErrI* observable is additionally defined, whose expression for minute m is as follows:

$$ErrI_m = \frac{\left\| \delta I'_{+(m)} - \frac{\delta I'_{+(m+1)} + \delta I'_{+(m-1)}}{2} \right| - \left| \delta I'_{-(m)} - \frac{\delta I'_{-(m+1)} + \delta I'_{-(m-1)}}{2} \right\|}{2}. \quad (16)$$

This observable helps to evaluate the symmetry degree of both polarizations, in such a way that a spike in both *disI* and *ErrI*-vs.-time plots in a given minute is a clear indication that the corresponding I value should be rejected. Similar expressions and procedures are valid for Declination, where a factor $\cos I_0$ is to be added in the denominator of Eqs. (11)–(14). As a real example, a spike is observed in the plot of the *ErrD* observable (Fig. 2, data from LIV observatory) after being detected in *disD*. If these tests had not been applied, this erroneous value would have passed unnoticed.

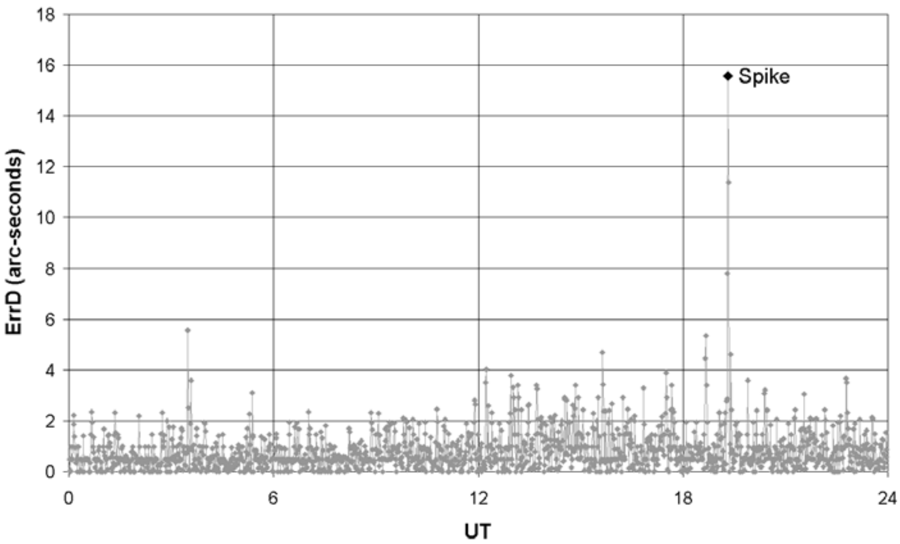

Fig. 2. A spike in Declination is corroborated by applying the *ErrD* observable. The value is clearly outside both the surrounding data samples and the instrument accuracy.

A simpler technique can be applied to detect spikes in total intensity, F. In this case, the observable *disF* can similarly be defined as $(F_1 - F_2)/2$, where both F_1 and F_2 are directly provided by the δDδI. This observable is also recommended to be computed for every minute and plotted against time providing a visual detection of erroneous values. Again, if the *disF* value for a given minute is clearly larger than both the proton magnetometer resolution and the natural magnetic disturbance for the surrounding minutes, it is a clear candidate for rejection.

2.5 Other errors

From Eq. (8) it can be concluded that the theoretical accuracy of the measurements is improved by increasing the magnetic fields induced by the coils (A_D and A_I). On the other hand, too great an applied field would produce an excessive gradient in the proton magnetometer sensor, resulting in a poor signal quality; therefore, an optimal value must be found. The finite dimensions of the coils, along with the possibility of the proton magnetometer not being exactly in the centre of both pairs of coils would also give rise to an increase in the magnetic gradient across the proton sensor. To avoid this, the BGS designed 80 cm diameter Helmholtz coils, generating a field A_x of circa 60% of the natural intensity, F. In this case, the gradient is such that a theoretical difference of a few tens of nT is produced between the centre and the extreme of the proton sensor (depending on its size). Other solutions to the gradient problems are the use of cylindrical windings, or the so-called spherical coils.

A significant temperature and humidity variation could result in a change in the δDδI configuration. For this reason, the thermal expansion coefficient of the material supporting the coils, as well as its water absorption capability must be low. For instance, the instrument in use at LIV has a theoretical surface thermal expansion coefficient of 1.9×10^{-5} K^{-1}, which implies no more than 0.2 mm yearly variation in the lineal dimensions.

3. Conclusions

The BGS δDδI has proved to be a reliable system, presenting some advantages with respect to other magnetometers, such as low temperature dependence and acceptable short-term stability. Other effects, such as long-term pier instabilities or distortion of the coils alignment can introduce additional errors if the measurements are not regularly corrected by absolute observations, this being the fundamental problem of some remote observatories and especially LIV, where absolute measurements can only be carried out for three months per year.

In examining the overall accuracy of the δDδI system we conclude that the main limiting factors have been found to be the ability of the coil current generator to reliably generate identical positive and negative bias currents, along with the temporal resolution being reduced by the long time period required to make a complete set of measurements.

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