

Repeat Station Data Reduction Using the CM4 Model

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Abstract

As an alternative to the commonly used method for the reduction of repeat station data, we have tested the relevance of the CM4 model for removing the external field. The dense network of observatories in Western Europe offers a good opportunity for this study. The prospect, however, is to elaborate a method which could be applied to the repeat station network of Madagascar, where only one observatory is available and the distance to the remotest repeat station is about 800 km.

1. Introduction

This study aims at improving the method of data reduction of repeat stations with application to the network of Madagascar. With the help of a nearby reference observatory, the classical method (Newitt *et al.* 1996) is based on the assumption that the transient variations of the magnetic field are identical at both the repeat station and the observatory. Under this assumption, the annual mean of the field at the repeat station S , centered on the date t of the measurement, is derived from the annual mean at the observatory O for the same time span by the equation:

$$\bar{\bar{B}}(S, t) = \bar{\bar{B}}(O, t) + (\bar{B}(S, t) - \bar{B}(O, t)) \quad (1)$$

The overhead bar stands for mean values, in this case, annual means. The error involved in the assumption of transient variation uniformity is difficult to assess. In addition, bearing in mind that a repeat station network is mainly devoted to internal field surveying, this simple method of reduction does not completely eliminate the external

field. Indeed, it is well known that annual means are still contaminated by long term external contributions which may be significant (i.e., Gavoret *et al.* 1986).

In an attempt to circumvent the drawbacks of the standard method, we have tried to model the external contributions by means of the comprehensive CM4 model (Sabaka *et al.* 2002, 2004) and to find out a formulae which could express the external field at any repeat station in an area around the reference observatory.

2. External Field Modeling

2.1 Preliminary

In order to validate the method, we have taken advantage of the dense observatory network in Europe. We have selected a set of observatories where data are available for the period 1993-2004. The observatory of Budkov (BDV), which is located in a more or less central position, has been chosen as reference observatory. The observatories THY, CLF, FUR, NCK, HRB, NGK, SUA play the role of repeat stations (Fig. 1a). Figure 1b displays an estimate of the error inherent to the classical method of reduction. The error is defined as the r.m.s. of $G(O,t) - G(S,t)$, after removal of the secular variation (fitted by a parabolic function of time). G stands for X, Y, or Z hourly means, O is the reference observatory, and S is one of the observatories of the set mentioned above. X, Y and Z are the field components in the geographical reference frame. As expected, the error increases with the distance to the reference observatory. In this example, it is about $5 \cdot 10^{-3}$ nT/km for X, $4 \cdot 10^{-3}$ nT/km for Y and $4 \cdot 10^{-3}$ nT/km for Z. (As a matter of comparison, the distance between TAN and the farthest repeat station of the network of Madagascar is about 800 km).

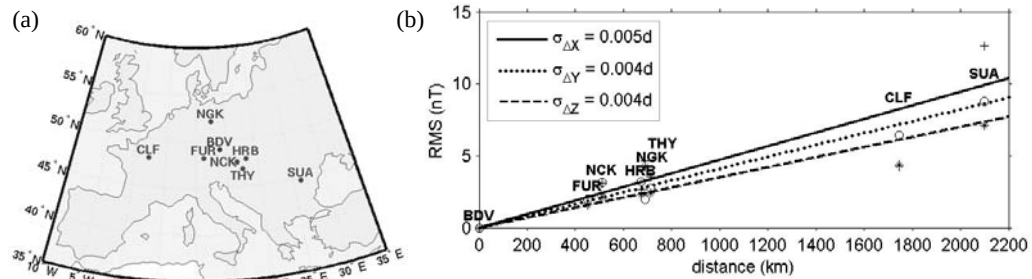


Fig. 1. (a) Location of the reference observatory (BDV) and the set of observatories used as repeat stations. (b) R.m.s. misfit of the hourly means (second term of right-hand side of Eq. 1) as a function of distance to the reference observatory, for the field components X (crosses), Y (circles), Z (stars), respectively.

2.2 Problem setting

The modeling outlined below deals with hourly means, in order to be consistent with the CM4 modeling. Figure 2 shows that, in the case of TAN observatory in par-

ticular, the CM4 model yields only a first approximation. In order to improve the modeling, we have to add second order terms. At the reference observatory, we suggest to model the field by the following equation:

$$\begin{aligned}\vec{B}(O, h) = & \vec{B}_{i,CM4}(O, h) + \vec{b}_i(O) + \delta\vec{B}_i(O, h) + \vec{B}_{e,CM4}(O, h) + ... \\ & \vec{b}_e(O, h) + \delta\vec{B}_{e,CM4}(O, h),\end{aligned}\quad (2)$$

where $\vec{B}(O, h)$ is the field measured at the reference observatory O (internal plus external); $\vec{B}_{i,CM4}(O, h)$ is the internal field given by the CM4 model at O ; $\vec{b}_i(O)$ is the bias field (various estimates of these bias fields have been published, i.e. Manda and Langlais, 2002; however, for the sake of consistency, the bias fields computed by CM4 have been adopted); $\delta\vec{B}_i(O, h)$ is the secular variation not modelled by CM4; $\vec{B}_{e,CM4}(O, h)$ is the external field given by CM4; $\vec{b}_e(O, h)$ is the long-term external field not modelled by CM4; $\delta\vec{B}_{e,CM4}(O, h)$ is the short-term external field not modelled by CM4.

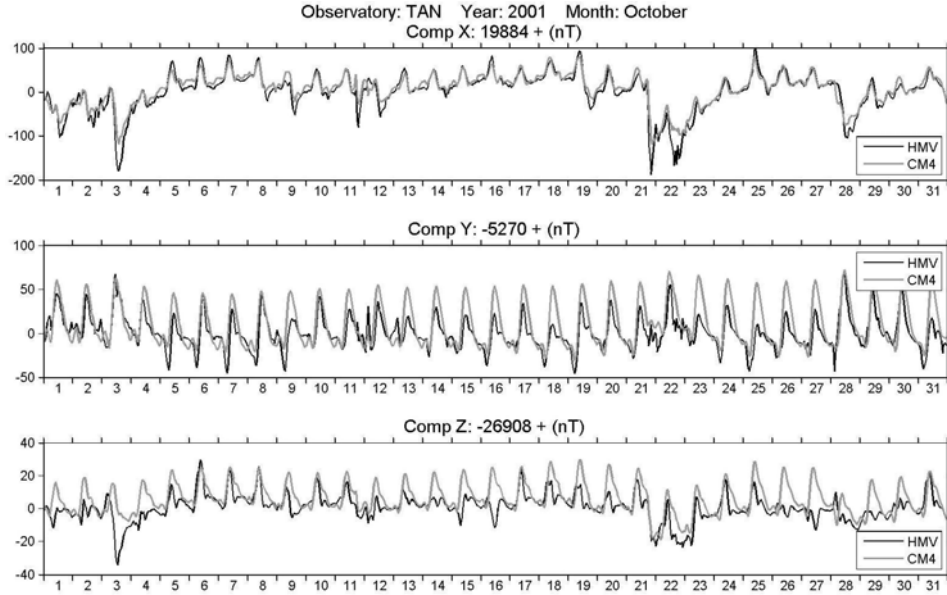


Fig. 2. Comparison of the hourly mean values (HMF) for TAN observatory and the modeling provided by the CM4 model for the month October 2001.

Let us define the residual field $\vec{\rho}(O, h)$ by:

$$\begin{aligned}\vec{\rho}(O, h) = & \delta\vec{B}_i(O, h) + \vec{b}_e(O, h) + \delta\vec{B}_{e,CM4}(O, h) \\ = & \vec{B}(O, h) - \vec{B}_{i,CM4}(O, h) - \vec{B}_{e,CM4}(O, h) - \vec{b}_i(O),\end{aligned}\quad (3)$$

where h stands for the time expressed in hours.

With respect to the aim of this study, terms $\vec{b}_e(O, h)$ and $\delta\vec{B}_{e,CM4}(O, h)$ are our main concern. In particular, we would like to construct $\delta\vec{B}_{e,CM4}(O, h)$ in such a way

that it is a stationary signal with annual zero mean. Figure 3 shows the distribution of $\rho(O, h)$ for the year 2001. Although the distribution may not be symmetrical (i.e. its mode different from its mean), the mean turns out to be the most robust measure of location. As $\delta\mathbf{B}_i(O, h)$ and $\mathbf{b}_e(O, h)$ vary smoothly with time, the distribution reflects mainly the statistical properties of $\delta\mathbf{B}_{e,CM4}(O, h)$.

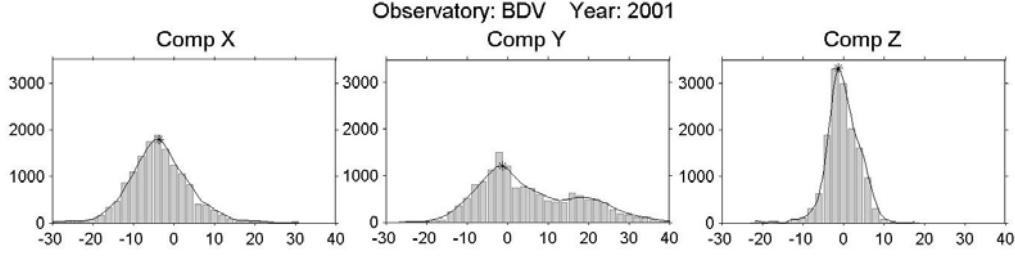


Fig. 3. Distribution of the hourly residual ρ in nT (Eq. 3) for each component and for the year 2001.

2.3 Residual secular variation and long-term external variations

Let $\bar{\bar{\rho}}(O, h)$ be the running mean value of $\bar{\rho}(O, h)$ over one year centered on h . Following Gavoret *et al.* (1986) we model the long term external variations using night time values in order to exclude most of the solar daily variation contribution. This selection is clearly equally valid for the identification of $\delta\mathbf{B}_i$. With the constraint $\delta\bar{\bar{B}}_{e,CM4}(O, h) = 0$, Eq. 3 leads to:

$$\bar{\bar{\rho}}(O, h) = \delta\bar{\bar{B}}_i(O, h) + \bar{\bar{b}}_e(O, h). \quad (4)$$

Figure 4b shows for each component X, Y, Z, the variation of $\bar{\bar{\rho}}(O, h)$ over the period 1994-2004.

We have now to model $\delta\bar{\bar{B}}_i(O, h)$ and $\bar{\bar{b}}_e(O, h)$. In the *CM4* model, the secular variation is modelled with a set of *B*-splines constructed with equispaced knots at 2.5 year intervals. Over the time span 1993-2004, it may equally well be modelled with orthogonal polynomials. Thus, the *CM4* internal field may be represented by:

$$\bar{\bar{B}}_{i,CM4}(O, t) = \sum_{k=0}^{K-1} \bar{\bar{a}}_k P_k(t). \quad (5)$$

The coefficients $\bar{\bar{a}}_k$ may be computed by a simple least-squares regression using the true *CM4* internal field as data. The regression yields estimates of $\bar{\bar{a}}_k$ as well as a covariance matrix.

$\delta\mathbf{B}_i(O, h)$ may be considered as a perturbation of $\mathbf{B}_{i,CM4}(O, h)$ likewise modelled with orthogonal polynomials restricted to low degrees. Accordingly, $\delta\bar{\bar{B}}_i(O, h)$ will be modelled with orthogonal polynomials. Their a priori coefficients $\bar{\bar{a}}_k^0$ as well as a covariance matrix may be derived from the corresponding parameters of $\delta\mathbf{B}_i(O, h)$.

According to Gavoret *et al.* (1986), we tentatively model $\vec{b}_e(O, h)$ (which, according to its definition, is close to $\mathbf{b}_e(O, h)$) as a function of time proportional to some index of geomagnetic activity or solar-terrestrial conditions. It turns out that the Wolf numbers are the most appropriate. Finally, $\vec{\rho}(O, h)$ is modelled by the equation:

$$\vec{\rho}(O, h) = \vec{\alpha} \bar{R}(h) + \sum_{k=0}^{K'-1} \vec{a}_k P_k(h) \quad (6)$$

where $\bar{R}(h)$ is interpolated from its smooth monthly values by interpolating splines.

The parameters $\vec{\alpha}$ and $\vec{a}_k$ are estimated by stochastic inversion. Figure 4b shows the model of $\vec{\rho}(O, h)$ computed according to Eq. (6).

2.4 Residual short-term external field

Having modelled $\delta \mathbf{B}_i(O, h)$ and $\mathbf{b}_e(O, h)$, we may return to Eq. (2), where the last unknown term is $\delta \mathbf{B}_{e, CM4}(O, h)$. $\delta \mathbf{B}_{e, CM4}(O, h)$ is assumed to be a small perturbation of the *CM4* external field model. Of the a priori parameters involved in the *CM4* modelling (conductivity model, solar flux $F_{10.7}$, Dst index), it turns out that the most easy to handle is Dst. Figure 4a shows that it is possible to fit exactly $\delta \mathbf{b}_{e, CM4}(O, h)$ by

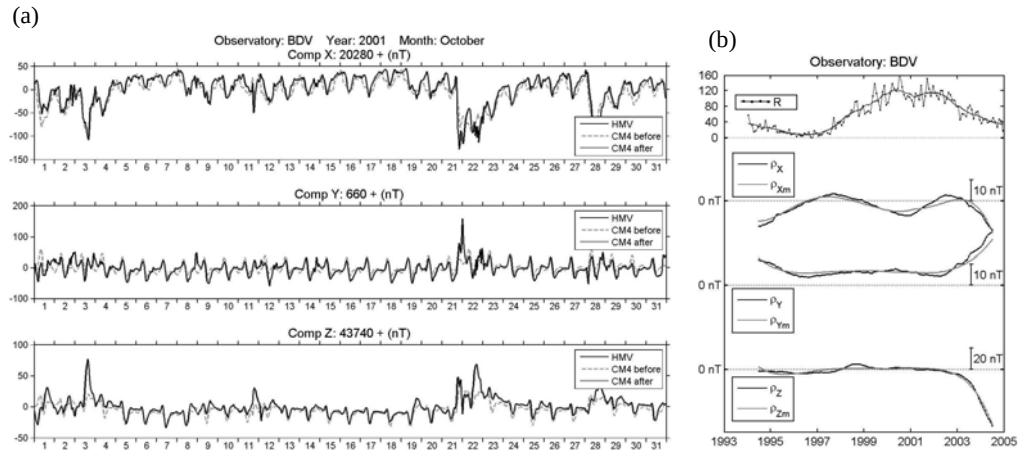


Fig. 4. (a) Example of exact modeling of $\delta \mathbf{B}_{e, CM4}(O, h)$ (Eq. 8) using hourly perturbations of the Dst index as adjustment parameter. Hourly mean data are for BDV observatory, in October 2001. See text for further explanation; (b) Black curves: variation of the running yearly mean value of $\boldsymbol{\rho}$ (Eq. 4) for each component X, Y, Z over the time span 1994-2004. Grey curves: modeling of $\boldsymbol{\rho}$ using Eq. (6).

a slight change δD_{st} of D_{st} (of course, δD_{st} has no longer the meaning of a geomagnetic index). On Fig. 4a, the adjusted curves cannot be distinguished from the actual ones. According to its definition, the field $\delta \mathbf{B}_{e, CM4}$ has a zero annual running mean. Let us recall that in the *CM4* model, D_{st} parameterizes only the dipolar term of the magnetospheric field.

Having computed, at the reference observatory, the perturbation δD_{st} required to obtain the best fit of $\delta \mathbf{B}_{e,CM4}(O, h)$, it is obvious to compute the same field $\delta \mathbf{B}_{e,CM4}(S, h)$ at any station S by the incorporation of δD_{st} into the $CM4$ external field model. Finally, our model for the external field is:

$$\vec{B}_e(S, h) = \vec{B}_{e,CM4}(S, h) + \delta \vec{B}_{e,CM4}(S, h) + \vec{b}_e(S, h). \quad (7)$$

The last step is the computation of $\mathbf{b}_e(S, h)$, which, at the moment, is only known at the reference observatory. We may assume that $\mathbf{b}_e(S, h)$ is the gradient of a time varying external magnetic potential. $\mathbf{b}_e(S, h)$ being known at only one point, only the dipolar term can be resolved unambiguously. Thus, we derive $\mathbf{b}_e(S, h)$ from the equation

$$\vec{b}_e(S, h) = -\overrightarrow{grad} \left[\frac{r}{a} (g_{e,1}^0(h) \cos \theta + g_{e,1}^1(h) \sin \theta \cos \varphi + h_{e,1}^1(h) \sin \theta \sin \varphi) \right]. \quad (8)$$

The Gauss coefficients $g_{e,1}^0$, $g_{e,1}^1$, $h_{e,1}^1$ may be easily derived from $\mathbf{B}_e(O, h)$ by standard computations taking into account the reference frame change from geodetic to geocentric.

3. Example and Discussion

Equation (7) yields the proposed formulae for the removal of the external field at any repeat station S . It is valid only within some area around the reference observatory. We may check its validity by comparison with the standard method. Figure 5a shows, for the year 2001, the “internal” field computed at CLF by means of the reduction equation:

$$\vec{B}_i(S, h) = \vec{B}(S, h) - \vec{B}_e(S, h) \quad (9)$$

where $\mathbf{B}_e(S, h)$ is computed with Eq. (7) as compared to $\mathbf{B}_i(S, h)$ computed with Eq. (1). The noisy pattern accounts for the imperfect removal of the external contribution. We observe an improvement on component Y only. The most conspicuous result is the nearly 20 nT difference in the X and Z components. This difference is not constant in time as shown in Fig. 5b where the monthly mean of the difference is displayed. Figure 6 is similar to Fig. 1a. It shows that the r.m.s. of the dispersion around a parabolic fit is nearly the same as for the standard method.

Despite the rather modest improvement in the scattering of the “internal” field, Fig. 5 shows, to our mind, an important, although well-known result: the annual mean rather imperfectly represents the internal field because it is still significantly contaminated by field of external origin. The $CM4$ -based modeling differs from the classical reduction of external field by a fluctuating non zero field. The fluctuations may account for the persistence of a long-term external component (Gavoret *et al.* 1986).

Improvements of this modeling which is a first order perturbation of the $CM4$ model can be easily imagined mainly along two lines. The first one would be to allow slight changes of the Gauss coefficients for the ionospheric field (Sabaka *et al.* 2002). around their $CM4$ values. The second would be to consider small changes of the conductivity model, which is valid essentially for Europe, in order to make the induced

part of the ionospheric field more flexible. Both improvements would require a deep scrutiny of the *CM4* model. In a recent paper, Olsen *et al.* (2005) suggest to split up the D_{st} index into an external and internal part. Although this new parameterization partly overlaps our empirical perturbation δD_{st} , it could balance or constrain changes of the ionospheric field. Improvements should also take into account the spatial distribution of the field $\mathbf{b}_e(S, h)$. The dipolar approximation is prescribed by the field being known at the reference observatory only. Although we believe that its geometry is simple, a worldwide analysis of observatory data could perhaps help to refine its description.

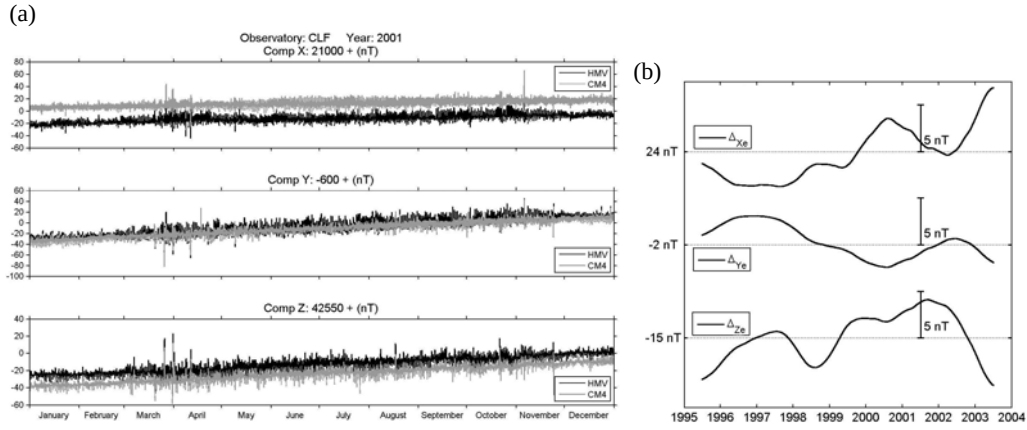


Fig. 5. (a) “Internal” field computed at CLF using Eq. (9) (curves labelled *CM4*) compared to the estimate given by Eq. (1) (curves labelled *HMV*); (b) Monthly mean of the difference (*CM4* modeling) – (*HMV* modeling) over the interval 1995–2004. The curves show that the difference is a function of time.

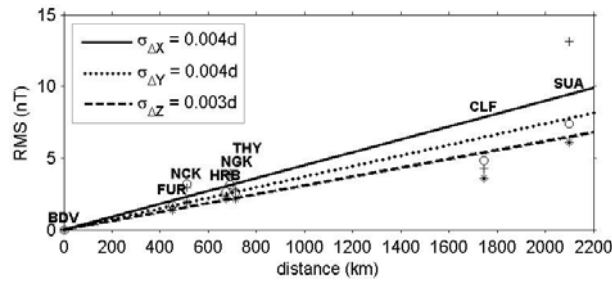


Fig. 6. R.m.s. misfit of the hourly means modelled by Eq. (9) as a function of distance to the reference observatory, for the field components X, Y, Z respectively. Same legend as Fig. 1 which it has to be compared to.

The last step, not considered here, would be to model field variation on the one-minute scale, which is more appropriate for the reduction of repeat station data. However, this step in the reduction process would be less critical either if the survey would

be carried out outside the main part of the daily variation, and during quiet days, or if the field variations would be recorded for a short time with a portable variometer.

4. Conclusions

The first order *CM4* model complemented by some second order terms derived from the analysis of the magnetic field variation at the reference observatory provides a promising tool for data reduction at the repeat stations. On one hand, with the second order modeling carried out in this study, its accuracy is only similar to the one of the standard method using a reference observatory. But, on the other hand, improvements of the external field modeling may be imagined, and may benefit from improvements of the *CM4* model or the like. The most conspicuous result is the mean difference between the “internal” field yielded by this method and the “internal” field (i.e. the annual mean) given by the standard method. The validation of the former with respect to the latter requires, however, further investigations. Finally, we have not tried to further reduce the field of the “repeat” stations to a common epoch. This should not be a serious drawback as regional modeling, like global modeling, can easily take into account measurements performed at different times.

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