Publications of the Institute of Geophysics, Polish Academy of Sciences
Geophysical Data Bases, Processing and Instrumentation
vol. 446 (A-32), 2023, pp. 35-41
DOI: 10.25171/InstGeoph_PAS_Publs-2023-010

Physicist in Search of the Theory of Earthquakes

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Abstract

Earthquake prediction is often considered the Holy Grail of seismology and earthquake science. However, earthquake research can be motivated both by challenge, when asking about seismic risk or hazard, or how lives could be saved, and by curiosity, when asking about the physics behind the observed patterns of seismicity. In this brief review, I focus on the second, cognitive aspect of earthquake science. I wonder how interesting seismicity and earthquake physics can be for a theoretical physicist turned seismologist.

1. INTRODUCTION

A physicist looks at the world through the prism of concepts such as momentum, energy, field, interactions, waves, entropy or probabilities, and instabilities. A geophysicist describes the Earth and the elements of its system, such as the geosphere, hydrosphere, cryosphere, atmosphere, or near-terrestrial space, in this way. A physicist turned seismologist applies the same tools of physics to study seismicity, earthquakes, and related processes.

There is no fundamental theory of earthquakes, such as the theory of electrodynamics or gravitation. There is a collage of diversity: specific models, concepts, or observations. Elastic rebound, earthquake cycle, slow and fast slips on faults, scaling relations, seismic moment budget, radiated seismic energy, stress trigger and shadow, slip deficit, and missing moment are the terms that we need to know to get some insight into the earthquake science. Thus, the earthquake science is more like the theory of turbulence, which remains 'the last unsolved problem of classical physics', though many of its specific problems have been well understood (Falkovich and Sreenivasan 2006). Understanding earthquakes and getting a consistent view on seismicity means putting all the pieces of the earthshaking jigsaw puzzle together (Hough 2002).

Mentioning that, which I believe is true, I start to think about Roman Teisseyre. I see him sitting at his desk and writing mathematical equations on a sheet of paper or making notes on a copy of a published research article. The search for a fundamental theory of earthquakes, similar to the theory of electrodynamics or gravitation, seems to be the main feature of his approach to his research, his Holy Grail of earthquake science. A physicist in search of the theory of earthquakes is how I remember him as a scientist, or as a theoretical physicist who became a seismologist.

2. SEISMOLOGY AND THEORETICAL PHYSICS

Gravitation. Einstein's theory of gravitation, or General Relativity, is based on the Einstein field equation, $\boldsymbol{G} = 8\pi\boldsymbol{T}$, where $\boldsymbol{G}$ is the Einstein tensor representing the space-time curvature, which can be expressed as a nonlinear function of the metric tensor, and $\boldsymbol{T}$ is the stress-energy tensor of the matter. Thus, the Einstein equation relates space-time geometry to the stress-energy of the matter (Misner et al. 1973). According to this view of the Universe, space-time is no longer merely a stage for events. On the one hand, matter and energy curve the space. On the other hand, the curvature of space determines the movement of matter and energy. Although this equation seems simple, it is in fact a complex system of non-linear differential equations with a wealth of applications and consequences. It governs the motions of luminous galaxies and dark matter, the evolution of the large-scale structure of matter in the Universe, and the geometry of spacetime, including instabilities, singularities, and gravitational waves caused by moving matter. By adding a term with the cosmological constant to the field equation, $\lambda\boldsymbol{g}$, where $\boldsymbol{g}$ is the metric tensor, you can account for dark energy in the Universe.

Earthquake physics is based on continuum mechanics, to which a similar geometrical approach can be applied, with the incompatibility law, $\boldsymbol{R} = \boldsymbol{S}$, playing the role of Einstein's field equation. Here, $\boldsymbol{R}$ represents the space curvature and $\boldsymbol{S}$ represents the sources of elastic strains and internal stresses in the medium (Kröner 1981). Dislocations or other defects, modelled as non-elastic or plastic, stress-free deformations within an elastic medium, play a similar role as matter in the theory of gravity, i.e., as sources of deformations and changes in the geometry of the medium. Since earthquake sources can be modelled as dislocation distributions, this view enables us to explain the deeper meaning of the seismic moment tensor, as well as interactions between tectonic faults, where a slip in one place can cause another slip in a distant place; or interactions among earthquakes, which can be treated as slip events on faults (see Appendix). Such a perspective allows the physicist to develop the theory of complex earthquake sources using similar mathematical tools, physical concepts, and intuitions, as has been done in the case of the theory of gravitation or other theories of physics, including electrodynamics.

Electrodynamics. We describe the electromagnetic field using the four quantities: electric field strength, $\boldsymbol{E}$, electric field induction, $\boldsymbol{D}$, magnetic field strength, $\boldsymbol{H}$, and magnetic field induction, $\boldsymbol{B}$. Electric charge density, $\boldsymbol{\rho}$, and electric current density, $\boldsymbol{J}$, describe the sources of the electromagnetic field and their motion. Maxwell's equations of the electromagnetic field determine the relationships between the quantities describing this field and its sources: Gauss' laws for electric and magnetic fields, and Faraday's and Ampère-Øersted's laws for, respectively, electric and magnetic induction (Jackson 1998).

The field equations are supplemented with three material equations characterizing the medium. The first defines the relationship between electric field strength and electric field induction, $\boldsymbol{D} = \varepsilon\boldsymbol{E}$, where ε is electric permittivity. The second defines the relationship between magnetic field strength and magnetic field induction, $\boldsymbol{B} = \mu\boldsymbol{H}$, where μ is the magnetic permeability. The third is Ohm's law, which defines the relationship between electric field strength and current density, $\boldsymbol{J} = \sigma\boldsymbol{E}$, where σ is conductivity. From Maxwell's equations, we obtain the laws of conservation of electric charge and electromagnetic field energy, i.e., the continuity equation and the Poynting equation, respectively.

Earthquake physics is based on elastodynamics, with the stress-free, plastic deformations to model seismic source dynamics (see Appendix). Respective theory can be formulated in analogy to electrodynamics. The stress, $\boldsymbol{\tau}$, stands for the electric field strength, $\boldsymbol{E}$; distortion, $\boldsymbol{\beta}$, stands for the electric field induction, $\boldsymbol{D}$. Displacement velocity, $\boldsymbol{v}$, stands for the magnetic field strength, $\boldsymbol{H}$, whereas momentum density, $\boldsymbol{w}$, stands for the magnetic field induction, $\boldsymbol{B}$. The field sources are the dislocation density, $\boldsymbol{\alpha}$, and the dislocation current density, $\boldsymbol{\rho}$. The field equations of elastodynamics have the same geometric interpretation as the equations of electrodynamics, including the equivalents of the Poynting equation and Ohm's law, which describe the strain and kinetic energy balance, together with its dissipation during slip (Bovet 1979). Although seismicity is a complex system and cannot be fully described by elastodynamics, its field equations enable us to gain deeper insight into various aspects of earthquake physics, including energy balance and interactions in seismic zones.

Cosmology. Galaxies, consisting of stars and interstellar gas clouds, can be seen with telescopes as "points of light". They are clustered in small and big groups, with hundreds to thousands of members. Such clusters are grouped in superclusters distributed along interconnected walls and filaments surrounding almost empty regions called voids (Martinez and Saar 2001; Gott 2016). The superclusters form a three-dimensional, large-scale structure of the Universe, which can be perceived as a cosmic web, with the richest galaxy clusters concentrated near its nodes. The observed galaxy distribution is the result of gravitational growth of small initial fluctuations or instabilities in the almost uniform early Universe (Peebles 1981).

However, the visible objects in the Universe are regarded as just tracers, or "light maps", of the underlying cosmic matter distribution. The galaxy clusters are above all dense concentrations of the dark matter, which can be detected by its gravitational effects and is thought to account for approximately 85% of the matter in the Universe. One of the objectives of the modern cosmology is to relate the visible objects, both optically and by X-ray observations, to the underlying, actual, mostly dark mass distribution (Borgani and Guzzo 2001).

Seismicity can be viewed in a similar way. The distribution of epicenters of strong earthquakes on the world map marks the boundaries of tectonic plates. Their patterns in specific regions, such as the Japanese subduction zone or the Nepal-Himalayan collision zone, reflect the complex processes underlying earthquake occurrence. For example, when we plot earthquake epicenters along the Japan Trench as circles with their radii related to earthquake source areas, for a given time period, we can recognize a kind of seismic web: a hierarchical structure with some visible voids, where earthquakes less frequently occur, as in the case of the cosmic web. That structure reflects the underlying plate interface physical features, such as asperities, where the plates are coupled stronger, and where higher stresses accumulate due to the plate movements, surrounded by weaker, non-asperity regions. Thus, earthquake sources reflect characteristic, deeper structures at the plate interface, like glowing galaxies reflect the deeper structure of dark matter in the night sky. The challenge is to recognize the nature of these voids: are they places of strong plate bonding, which, when cracked, become the source of strong earthquakes; or, on the contrary, are they places of weak bonding, where the slip is slow, invisible to seismometers.

Turbulence. Turbulence can be defined as a chaotized flow. It can be viewed, therefore, from two different points of view. First, as the flow, it is modelled by the Navier-Stokes equations, which result from Newton's second law applied to the viscous fluid motion and express momentum balance and conservation of mass in hydrodynamics. Second, as the chaotic process, which is irregular in both time and space, it can be described in terms of statistical averages, such as moments of velocity differences with their dependences on spatial scales (Falkovich and Sreenivasan 2006).

Mechanism for the generation of turbulent flows can be considered as an energy cascade. The energy is injected at the largest length scale; it is transferred or cascaded without loss through the intermediate, so-called inertial range of scales, and it is finally dissipated by viscosity at its characteristic, smallest length scale. Nonlinear effects are responsible for producing a hierarchy of motions on smaller and smaller scales.

The motions of individual particles in a turbulent flow are irregular and unpredictable, but statistical tools enable us to reveal some order, or geometrical and dynamical structures, that occur in such a complex process. They are expressed as statistical conservation laws and scaling relations, which describe how different statistical averages change with scale or size measures.

Seismicity is driven by the slow relative movement of tectonic plates, and the energy thus supplied to the system is released through fast and slow slips along faults, so aseismic creep phenomena, slow slip events, and earthquakes, all of varying sizes and scales.

The earthquake rupture processes can be simulated by using the differential-integral equation, where a slip velocity at a given point on a fault depends on the net stress, so the driving stress caused by the tectonic loading and slips or deformations in the whole medium, minus the frictional stress due to material characteristics of the fault at that point (Senatorski 2019). However, the seismicity models should involve probabilistic principles that reflect our uncertainty about the future earthquakes. By looking at the geodetic and seismic data, it is easy to explain why a given large earthquake did occur at a given time and site, and why its magnitude and other characteristics are as observed, but we do not know those details in advance. What we can only guess are possible scenarios or outcomes. We do not have enough knowledge to predict which of the possible scenarios will be realized, though we do know the constraints they should meet.

Such constraints can be learned from energy budget observations, accumulated and released strains due to, respectively, the tectonic plate movements and earthquakes, or alternatively, slow slip events observed in a given region. They result also from the plate interface characteristics, which are reflected by statistical scaling relations among earthquake parameters, and from our understanding of the relation between the structure and material characteristics of a subduction channel at the plate interface and the plate coupling strength at a given location (Senatorski 2020). We should use all and only available knowledge for the honest and effective forecasts of the largest earthquakes, so that our uncertainty about the system outcome is also taken into account. In a sense, the slippage occurring on a fault can, as in the case of turbulence, be called the chaotized flow; and that is why its modeling should take into account both our knowledge and uncertainty.

3. CONCLUSIONS

Earthquake prediction is often considered the Holy Grail of seismology and earthquake science. However, earthquake studies can be motivated both as a challenge to save lives and by curiosity about earthquake processes. This short review is focused on the second aspect, when we ask about the physics behind the observed seismicity patterns.

Referring to four examples – the theory of gravity, electrodynamics, cosmology, and turbulence – I tried to show that seismology, as a part of geophysics, can and should draw inspiration from even such distant fields of physics. I think that field theory, statistical physics, and astrophysics, along with geophysical observations, were the main inspiration and starting point for Roman Teisseyre's considerations reflected in his books and articles (Teisseyre and Teisseyre-Jeleńska 2014). Among these inspirations, the continuum dislocation theory, briefly discussed in the appendix, played a special role.

Seismicity is a complex system. Seismic networks and space geodesy provides a detailed view of the stress accumulation-release process at the plate interface in seismic zones. This approach has not brought effective earthquake predictions. We hope that new methods of data processing, including machine learning methods, will help to solve the problem. But perhaps we also need more creative and more imaginative theoretical considerations – and perhaps surprisingly simple solutions – as is the case in other fields of modern physics. What makes the science of earthquakes so fascinating is that its different aspects – cognitive and practical – cannot be separated.

APPENDIX

Earthquakes are understood as processes of fast slips along faults. An earthquake source can be modelled as a plastic deformation within an elastic medium representing the crust of the Earth. Elastic deformations are related to stress by Hook's law. Plastic deformations are stress-free.

Thought experiment. The following procedure can be imagined (Kröner 1981). A macroscopic body is built up of many small-volume elements. Each of the small elements is deformed plastically, so that they do not fit well together. Then, we deform them elastically in such a way that they fit together again. The elastic deformation should be just opposite to the plastic one to do the task. After the elements are welded and the forces responsible for the elastic deformation are removed, the body relaxes into the lowest energy state by a second elastic deformation.

The same procedure can be applied to a finite earthquake source volume and its surroundings. First, we cut out and remove the volume from its neighborhood and deform it plastically. Then, we deform the separated volume elastically so that, when inserted back into its surroundings, it fits into the empty space. When the entire medium relaxes, additional elastic deformation occurs both in the source volume, which was plastically deformed, and outside the source volume, where there was no plastic deformation.

These imagined procedures can be described mathematically by using plastic, $\boldsymbol{u}^P$, elastic, $\boldsymbol{u}$, and total, $\boldsymbol{u}^T$, displacement fields, and related distortions $\boldsymbol{\beta} = \nabla\boldsymbol{u}$, $\boldsymbol{\beta}^P = \nabla\boldsymbol{u}^P$, and $\boldsymbol{\beta}^T = \nabla\boldsymbol{u}^T$, so that $\boldsymbol{u}^T = \boldsymbol{u} + \boldsymbol{u}^P$ and $\boldsymbol{\beta}^T = \boldsymbol{\beta} + \boldsymbol{\beta}^P$. To see how stresses are generated, symmetric elastic, plastic, and total strain tensors are defined as symmetrized gradients of the displacement vector fields, respectively, $\boldsymbol{\varepsilon} = \nabla_S\,\boldsymbol{u}$, $\mathrm{d}\boldsymbol{\varepsilon}^P = \nabla_S\,\boldsymbol{u}^P$, and $\boldsymbol{\varepsilon}^T = \nabla_S\,\boldsymbol{u}^T$, so that the total strain tensor is $\boldsymbol{\varepsilon}^T = \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}^P$.

Dislocations and incompatibility law. In general, there is no plastic or elastic unique displacement fields in the welded macroscopic body, so neither $\boldsymbol{\beta} = \boldsymbol{\beta}^T - \boldsymbol{\beta}^P$ nor $\boldsymbol{\beta}^P$ can be expressed as gradients of the dis-placement fields. Instead, they can be defined as differential forms, $\boldsymbol{du} = \boldsymbol{\beta dx}$ and $\boldsymbol{du}^P = \boldsymbol{\beta}^P\boldsymbol{dx}$, where $\boldsymbol{du}$ represents a change of a distance between points that were at a distance $\boldsymbol{dx}$ before the deformation. Distortions $\boldsymbol{\beta}$ and $\boldsymbol{\beta}^P$ are incompatible, if they are not compatible with the existence of a unique displacement fields and may not be expressed as gradients. The total distortion has to be compatible, as far as the body is not broken into pieces. This means that its rotation vanishes, $\nabla \times \boldsymbol{\beta}^T = \nabla \times \nabla\boldsymbol{u}^T = 0$. However, $\nabla \times \boldsymbol{\beta}^P = \boldsymbol{\alpha} \neq 0$, so for elastic distortion we have the first incompatibility law:

$$\nabla\times\boldsymbol{\beta} = -\boldsymbol{\alpha}\ ,$$

with the dislocation density tensor, $\boldsymbol{\alpha}$, as a source of elastic distortion (Kröner 1981).

For strains, the incompatibility tensor is then defined as $\boldsymbol{\eta} = \nabla \times \boldsymbol{\varepsilon}^P \times \nabla$ (meaning that rotation is calculated column-wise then row-wise). It is equal to zero, if $\boldsymbol{\varepsilon}^P$ can be expressed as a symmetrized gradient of the plastic displacement field, $\boldsymbol{\varepsilon}^P = \nabla_S\,\boldsymbol{u}^P$. Since for the total strain tensor we have $\boldsymbol{\varepsilon}^T = \nabla_S\,\boldsymbol{u}^T$, the second incompatibility law is:

$$\nabla \times \boldsymbol{\varepsilon} \times \nabla = -\boldsymbol{\eta}\ ,$$

with the dislocation incompatibility tensor, $\boldsymbol{\eta}$, as a source of elastic strains (Kröner 1981).

The second incompatibility law provides us with the geometric constraints on the elastic strain field, $\boldsymbol{\varepsilon}$, if sources $\boldsymbol{\eta} \neq 0$ are present. This local formula does not change when the elastic deformation described by any displacement field, $\boldsymbol{\lambda}$, is imposed. Any solution of the incompatibility law may be written as $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}' + \nabla\boldsymbol{\lambda}$, where $\boldsymbol{\varepsilon}'$ is its particular solution. The compatible strains $\nabla\boldsymbol{\lambda}$, which can be produced by external forces, do not change the geometric constrains given by the incompatibility law. Using this invariance, the incompatibility law can be rewritten, in analogy to the Einstein's field equations, as $\boldsymbol{R} = \boldsymbol{S}$. Tensor field $\boldsymbol{R} = \Delta\boldsymbol{\varepsilon}$ represents space curvature and can be expressed as function of the metric tensor, whereas tensor field $\boldsymbol{S} = ½\,(\boldsymbol{\eta} - \eta\hat{I})$, where η is the trace of the incompatibility tensor, represents sources of elastic strains and internal stresses in the medium.

The incompatibility tensor can be expressed as symmetrized rotation of the dislocation density tensor, $\boldsymbol{\eta} = (\nabla \times \boldsymbol{\alpha})_S$. For given internal stress sources $\boldsymbol{S}$, elastic strains can be calculated by using Green's function, as in the case of theory of gravitation.

Seismic moment tensor. The seismic moment tensor is one of the basic quantities characterizing the size and strength of a seismic source. Based on the scalar seismic moment, the so-called earthquake magnitude, m_W, is defined. The physical interpretation of the seismic moment is based on the concept of plastic deformations, i.e., inelastic, free of stress deformation of the medium within the seismic source volume (e.g. Udías et al. 2014; Madariaga 2015).

After the procedure described above, elastic strains are $\boldsymbol{\varepsilon} = \nabla_S \boldsymbol{u}^T - \boldsymbol{\varepsilon}^P$, where $\boldsymbol{\varepsilon}^P = 0$ outside the seismic source volume. The related stress field is, according to Hook's law:

$$\boldsymbol{\sigma} = \boldsymbol{C}\cdot\boldsymbol{\varepsilon} = \boldsymbol{C}\cdot\nabla\,\boldsymbol{u}^T - \boldsymbol{m}\ ,$$

where the seismic moment density tensor, $\boldsymbol{m} = \boldsymbol{C}\cdot\boldsymbol{\varepsilon}^P$, is the excess stress, i.e., the difference between the stress due to displacements $\boldsymbol{u}^T$ and Hook's law, and the actual stresses in the medium. $\boldsymbol{C}$ is the elasticity or stiffness tensor.

The tensor field $\boldsymbol{m}(t, \boldsymbol{r}) = 0$ outside the source region. Its gradient determines the additional volumetric force to be included in the equations of motion for a continuous medium. Integrating over the source area, we get the tensor of the total seismic moment of the source:

$$\boldsymbol{M}(t) = \iiint d\boldsymbol{r}\ \boldsymbol{m}(t, \boldsymbol{r})\ ,$$

which can be interpreted as the internal stress generated in the medium by its inelastic deformation within the seismic source.

In the case of slip along a fault, displacement discontinuities are obtained by passing to the zero limit of the thickness, h, of the narrow zone of plastic deformation in such a way that the volume integral of deformations remains finite. We write the deformation tensor as a generalized function:

$$\lim_{h\to 0} \boldsymbol{\varepsilon}^P(t, \boldsymbol{r})\, h = [\Delta\boldsymbol{u}\ (t, \boldsymbol{r})\ \boldsymbol{n}]_S\ \delta(\boldsymbol{r} \in A)\ ,$$

where $\Delta\boldsymbol{u}$ denotes slip along the plane A, and $\boldsymbol{n}$ is the unit vector normal to A.

The seismic moment density tensor is:

$$\boldsymbol{m} = \boldsymbol{C}\cdot\Delta\boldsymbol{u}\cdot\boldsymbol{n}\ \delta(\boldsymbol{r} \in A)\ ,$$

and the total seismic moment is:

$$\boldsymbol{M}(t) = \iint dA\ \boldsymbol{C}\cdot\Delta\boldsymbol{u}(t, \boldsymbol{r})\cdot\boldsymbol{n}\ .$$

For isotropic medium $C_{ijkl} = \lambda\delta_{ij}\,\delta_{kl} + \mu(\delta\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk})$, so the seismic moment tensor can be written as:

$$\boldsymbol{M}(t) = 2\mu\overline{D}A[\boldsymbol{n}\cdot\boldsymbol{d}]_S\ ,$$

where $M_O = \overline{D}\mu A$ is the scalar seismic moment, μ is shear modulus, $\overline{D}$ is mean slip, and $\boldsymbol{d}$ is the slip unit vector.

Other applications. Both discrete (Anderson et al. 2017) and continuum (Kröner 1981) theories of dislocations allow to model complex, heterogeneous seismic sources, including their static characteristics and long-range interactions, rupture dynamics, as well as aseismic processes leading to earthquake rupture. Their generalization to a more complex medium with asymmetric stresses, or non-symmetric elasticity tensor, rotational effects, and conjugated strain modes enabled Roman Teisseyre to develop his innovative approach towards earthquake source physics (e.g. Teisseyre and Teisseyre-Jeleńska 2014).

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Received 18 September 2023
Received in revised form 13 October 2023
Accepted 22 November 2023