Publications of the Institute of Geophysics, Polish Academy of Sciences
Geophysical Data Bases, Processing and Instrumentation
vol. 446 (A-32), 2023, pp. 43-48
DOI: 10.25171/InstGeoph_PAS_Publs-2023-011

A Short Story of My Search for Rotational Waves – from the Asymmetric Theory to DEM Simulations

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Abstract

In this short note, I present preliminary results of the simulation of rotational waves generation in a homogeneous amorphic material using the Discrete Element Method approach. It is demonstrated that in this type of material, the classical, transient rotational waves essentially do not exist, but the rotational energy is generated and accumulated near the free surfaces of fractures. This observation opens a question about the role of rotational effects in the total energy balance of the earthquake generation process.

1. INTRODUCTION

Rotational waves in solid media have been an intriguing issue for physicists for years; see, e.g. Kozak (2009). Within the classical linear theory of elasticity, such waves – transient propagation in time and space of rotational energy – do not exist. The reason is that the linear elasticity theory assumes that the considered perturbations (e.g. waves) are small in magnitude so the strain tensor is fully symmetric (Aki and Richards 1985) and a linear relation between stress and strain (Hook's law) can be assumed. In consequence, no rotational movement is supported by the theory. However, there exist more complex theories of elastic solids, like, for example, the micro-polar one (Nowacki 1986; Cosserat and Cosserat 1909) in which rotational waves can actually exist. This fact has been completely ignored in seismology for years, because in the course of its development the main effort had been directed towards an accurate description of mechanical (seismic) waves in so complex and heterogeneous medium as the Earth. In other words, the real problem in seismology was an accurate modeling of gross features of seismic waves and all "minor effects" have been put aside. Roman Teisseyre was probably the first person who raised the problem of rotational waves in a seismological context (Abreu et al. 2017). Through the years he has been continuing an effort of building a consistent theory of asymmetric continuum medium with the hope it can enrich seismology by a new tool to study the Earth and processes within it. At the beginning, his effort was treated mostly as an interesting, but only theoretical adventure because there was no observational evidence of such rotational seismic waves. Actually, there was no evidence because nobody was interested in looking for such effects. Moreover, there was no available equipment that could measure such subtle waves. The situation started to change when observations of damages caused by earthquakes definitely proved the existence of rotational movements in epicentral areas (see, e.g. Cochard et al. 2006; Takeo 2006). From that time on, the interest in rotation waves in seismology has raised and rotational seismology has emerged.

When I came back to the Institute of Geophysics after 3 years spent with Prof. A. Tarantola at the Institute de Physique du Globe in Paris, I immediately encountered Roman Teisseyre and his "seismic rotational waves" issue. Being devoted to the inverse theory and its seismological application I have treated Teisseyre's attempt of building asymmetric theory predicting the rotational waves with some distance. However, many discussions and a minor help with numerical methods I gave him slowly raised my interest in this issue. Thus I have started to study his theory. Following his ideas of asymmetric continuum hidden in complex mathematical formulas and continuously changing notations was extremely difficult. What I have finally recognized, however, is that the existence of rotational waves, no matter how mathematically represented (e.g. by asymmetric stress tensor) requires an existence of "hidden degrees of freedom" in the medium. Fortunately, that time I have started numerical simulations using the Discrete Element Method – the method which by construction represents the medium by an ensemble of independent but interacting elements. The fact that these elements have not only the translational degree of freedom (the fact used in the classical theory of elasticity) but also rotational ones ("hidden degree of freedom") was an ideal starting point for analysing the rotational waves. I have quickly recognized this adventure of the DEM method and together with my former students (Natalia Foltyn, Alicja Kosmala, and Piotr Klejment) we have started some simulations of rotational waves. Unfortunately, it took much time to get some reasonable results so that I could not communicate them to Roman Teisseyre. I describe them here in a very short form to memorize how Teisseyre's devotion to this task inspired me to make a closer look at the rotational waves issue.

2. DEM SIMULATION OF ROTATIONAL WAVES

The discrete element method is a numerical tool originally designed to describe the behaviour of granular media (Cundall 1971, Potyondy and Cundall 2004). Its distinguishing feature is representing a medium in hand by a set of independent particles which can interact with each other. Since elements have a finite size, both translation and rotational movements are possible and time evolution is described by a direct application of the Newton (translational movement) and Euler (rotation) equation (Abe et al. 2014). Thus, the method offers a very elementary approach to dynamic simulations without any additional assumptions. The only information we need to provide is the rule of interactions between particles. For this reason, the method is very well suited for studying the rotational waves issue.

Using this method we have started from the simplest question of whether we can observe excitation of rotational degree of freedom in realistic conditions. To answer this question we have designed an experiment shown in Fig. 1. A thin plate built of bounded spherical particles was used as a model of amorphic solid material. At the beginning of simulation, one of the particles located near the centre of the plate starts to move right with a constant velocity. Its movement breaks some inter-particle bonds, creating a horizontal fracture and collision with other particles to transmit energy to the plate body. In such a configuration, "off-fault" secondary cracks are also created. Since the particle-particle interaction model allows for the existence of tangential forces in non-central collisions, which in our case dominate the energy transfer process, both translational and rotational energy is built up. In Fig. 2, a few snapshots of the process are shown with a separate presentation of translational (left column) and rotational (right column) energy. What is apparently visible is that the translational energy takes

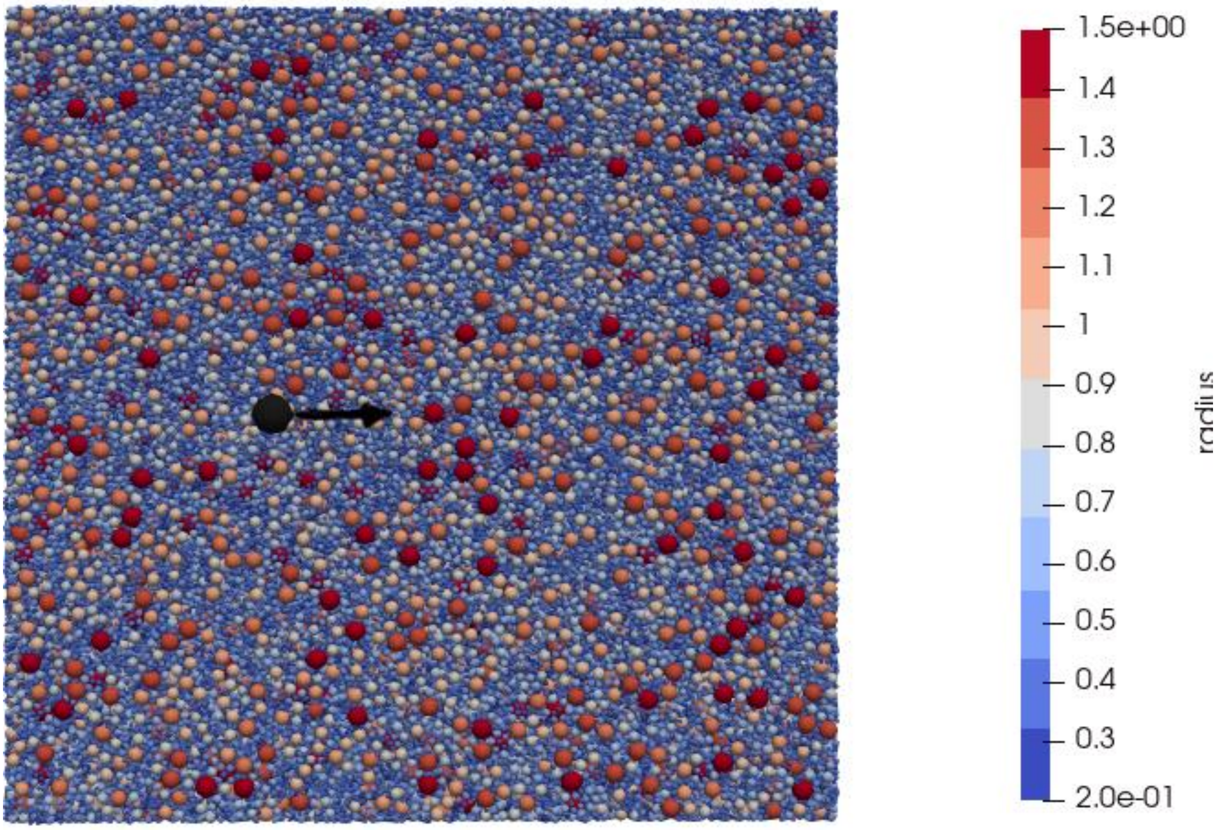

Fig. 1. Sketch of a numerical simulation setup designed for an analysis of rotational waves. The 5 cm × 5 cm × 6 mm "rock" sample has been built of spherical particles with radii in the range 0.2–1.2 mm. One of the particles (marked by black) starts to move right at the beginning of the simulation with a predefined velocity transferring energy to the plate.

the form of waves which propagate through the medium, as expected. The evolution of rotational energy seems to be different. It apparently concentrates around boundaries of both the main fracture as well as secondary, "wing" fractures and it essentially does not form the classical transient waves.

The conclusion of our simulations is by no means clear. We have essentially observed no running rotational waves. However, we have apparently observed a building of rotational energy in the plate. Although at first sight, this observation seems to be negative, it is really not the case. The observed spatial distribution of rotational energy can have an important impact on our understanding of the earthquake generation process. The point is that the whole classical analysis of earthquake mechanisms does not include the rotational energy in the energy budget of the earthquake (Cocco et al. 2023). However, Fig. 3 shows that such energy is comparable with the linear kinetic energy at the nucleation stage. Moreover, it concentrates around fracture surfaces where most of dynamical breaking process occur. The concentration of rotational energy around secondary "off-fault" fractures opens a question on its role in the creation of off-fault damages, well observed in real cases (Okubo et al. 2019). Currently, we have no answer to these questions. The dependence of the obtained results on the structure of the plate, including the size of the particles used and the way of energy excitation in the plate should also be analysed.

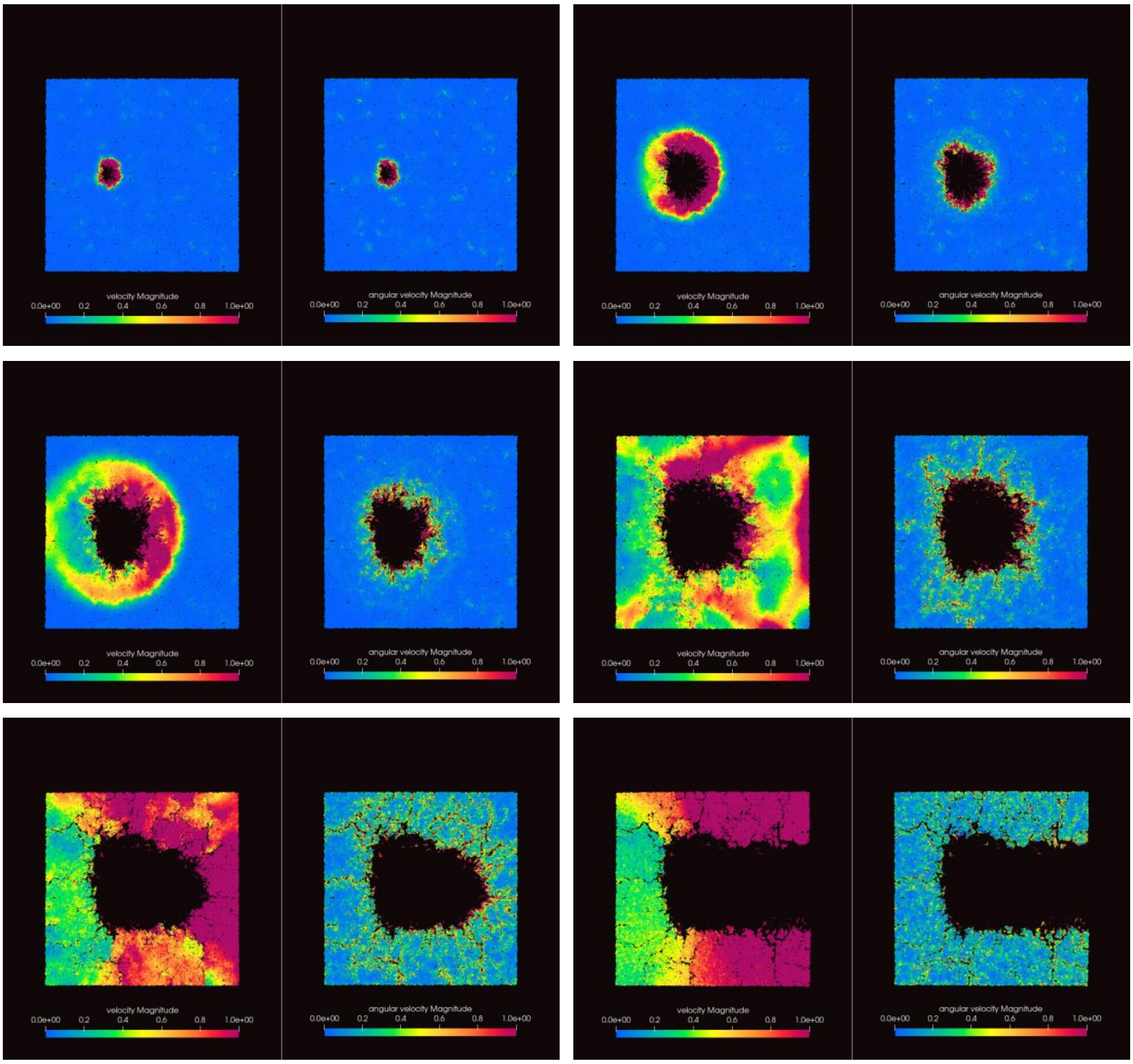

Fig. 2. Snapshots of linear and rotational velocity of particles building the sample. Each snapshot consists of two panels: left showing the linear velocity and right with rotational velocity, respectively. The snapshots correspond to simulation times 10 ms (left top), 30 ms (right top), 50 ms and 70 ms (middle row) up to 100 ms and 130 ms (lower row). Some inter-particle bonds are broken due to a movement of the "source" particle leading to creation of the main, horizontal crack and secondary, "wing" cracks. The formation of a classical (translational-type) acoustic wave is clearly visible at $T = 30$ ms and $T = 50$ ms. At later stages of the simulation, it undergoes a classical reflection from the boundaries of the sample ($T = 100$ ms). On the other hand, the rotational energy concentrates along the created free surfaces of cracks and does not form any visible transient wave-train.

3. CONCLUSION

The analysis presented here would have never been undertaken if not a curiosity of Roman Teisseyre for a deeper understanding of the surrounding world. Although his ambition of developing the theory of asymmetric continua has gone much further than seismology (we were even discussing gravitational theories) he has apparently opened a new branch of seismology – the rotational seismology. The questions which I have formulated at the beginning of this short

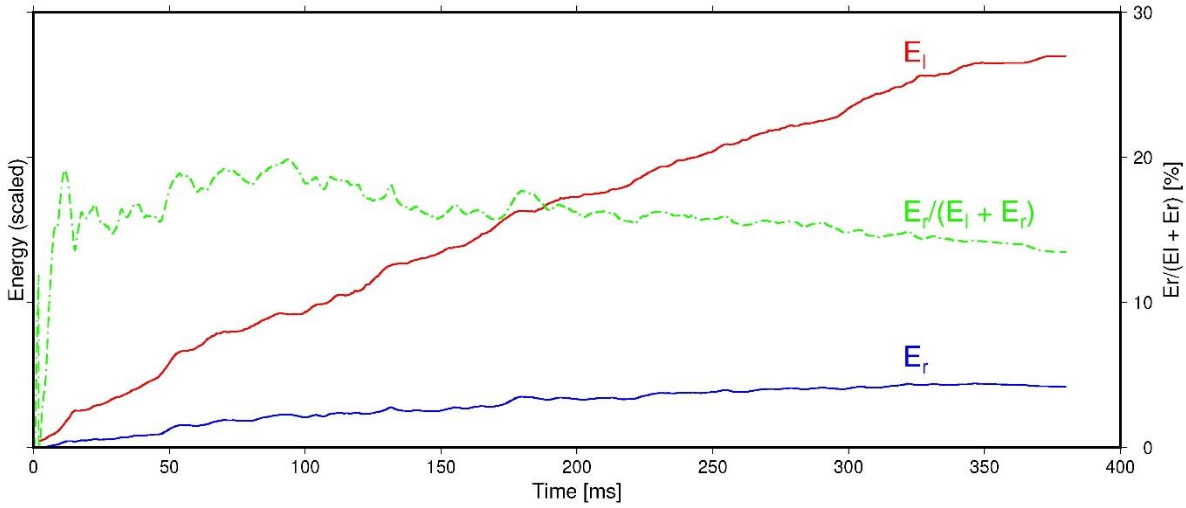

Fig. 3. Kinetic energies of linear (E_l – red line) and rotational (E_r – blue line) movements of particles. The ratio of rotational kinetic energy to the total kinetic energy ($E_T = E_l + E_r$) is shown in green. Let us note that during the initial phase of the rupture development, this ratio reaches a considerable value of 20% which means that in this stage the rotational effects are quite important.

essay and which follow from the simple numerical experiments are intriguing. I have no doubt that sooner or later they will find their proper analysis and satisfactory answers within the framework of the earthquake mechanics. I want to emphasize here that they are direct consequences of many discussions with Roman Teisseyre and my problems with a full understanding of the asymmetric continua theory which he was developing through many years of scientific activity.

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Received 19 July 2023
Received in revised form 22 November 2023
Accepted 23 November 2023